MATH 665: TOPICS IN QUANTUM ALGEBRA

FALL 2024 SYLLABUS

We discuss the relationship between representations of linear groups over finite and p-adic fields, a part of Lie theory, and isotopy invariants of knot and links, a part of geometric topology. The bridge is the theory of Hecke algebras and their cocenters.

Instructor	Minh-Tâm Trinh (minh-tam.trinh@yale.edu)
Time	TTh 2:30–3:45 PM
Place	17 Hillhouse Ave, Room 03 (basement) (**NEW**)
Webpage	https://mqtrinh.github.io/math/teaching/yale/math-665/

In place of a textbook, I will typeset course notes and post them to the webpage as we go along. See also the bibliography at the end of this syllabus.

Schedule

8/29	Introduction	
9/3–9/5	1. Finite Reductive Groups	Set 0 (*due* 10/10) Set 1 (*due* 9/19)
9/10–9/12		
9/17–9/19		
9/24–9/26	2. Hecke Algebras and Link Invariants	Set 2 (*due* 10/17)
10/1–10/3		
10/8–10/10		
10/15–10/17	3. Categorification, *October Recess*	
10/22–10/24		Set 3 (*due* 11/14)
10/29–10/31		
11/5–11/7	4. Current Topics	
11/12–11/14		
11/19–11/21		Set 4 (*due* 12/5)
11/26–11/28	*November Recess*	
12/3–12/5		

Logistics

Emails. If you need to email me about the course, please put "MATH 665" in the email subject. That helps me keep everything organized. You may address me as "Minh-Tam" or as "Dr. Trinh".

Grades. Problem Set 0 is only assigned to the undergraduates enrolled in the course. If you are taking the course for a grade (in any role), then:

- To pass the course, you must earn points on at least one problem set.
- To get a B-range grade or higher, you must earn more than one-third of the total possible points across all problem sets (including Problem Set 0 if applicable).
- To get an A-range grade, you must earn more than half of the total possible points (including Problem Set 0 if applicable).

You should write your homework in complete sentences.

There is no attendance grade. If you get sick, please stay at home and take care of yourself.

References

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[C2] R. W. Carter. I. On the Representation Theory of the Finite Groups of Lie Type over an Algebraically Closed Field of Characteristic 0. In *Algebra IX*. Ed. A. I. Kostrikin & I. R. Shafarevich. Encyclopaedia of Mathematical Sciences, Vol. 77 (1996), 1–120.

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