MATH 430: INTRODUCTION TO TOPOLOGY
MIDTERM GUIDE

SPRING 2025

The midterm exam will be held in-class on **Monday, February 24**. It will start 1–2 minutes after the start of class time (11:35 am). It will be a closed-book exam, designed to take < 70 minutes.

What Could Appear.

§12.

- definitions of a topology and of topological spaces
- what it means for one topology to be finer than another
- definitions of the discrete, indiscrete, and finite-complement topologies on any set, and why these are topologies
- definitions of the analytic and lower-limit topologies on $\mathbf{R}$, and why these are topologies

§13.

- what it means for a collection of sets to be a basis or subbasis for some topology
- what it means for a subbasis to generate a given basis, or for a basis to generate a given topology
- examples where different bases generate the same topology

§16.

- definition of the subspace topology on a subset of a topological space
- (multiple) examples $Y \subseteq X$ where some $U \subseteq Y$ is open in Y, but not in X; and a condition that rules out such situations (Problem Set 1, #8)

§17.

- definitions of closed sets, interiors, closures, limit points, convergence
- given $A \subseteq Y \subseteq X$, how the interior/closure of A in Y are related to the interior/closure of A in X
- relationship between closures and limit points
- what it means to check that a space is Hausdorff ($= T_2$) or T_1
- examples of spaces which are T_1 but not Hausdorff, or which are not T_1
- relationship between the Hausdorff axiom and limit points

§18.

- what it means for a map between topological spaces to be continuous
- what it means to "check continuity using a basis/subbasis of the target"
- what it means for a continuous map to be a homeomorphism, or for topological spaces to be homeomorphic
- examples of homeomorphisms between distinct sets (*e.g.*, $\mathbf{R}$ and $(0, 1)$)

- examples of continuous bijections that are not homeomorphisms
- the pasting lemma (Munkres Thm. 18.3)

§15, 19.

- definitions of (cartesian) products and projection maps
- definitions of the box and product topologies on $\prod_\alpha X_\alpha$
- characterization of the product topology in terms of continuity (Munkres Thm. 19.6)
- examples of bases for the box and product topologies on $\mathbf{R}^\omega$
- properties of and differences between the box and product topologies on $\mathbf{R}^\omega$ (Problem Set 3)
- definition of $\mathbf{R}^\infty \subseteq \mathbf{R}^\omega$

§20.

- definition of metrics and the topologies they generate
- why the open balls for a metric form a basis for the associated topology
- why the euclidean and square metrics both generate the analytic topology on $\mathbf{R}^n$
- comparison of the uniform topology on $\mathbf{R}^\omega$ to the box and product topologies

§22.

- definition of the quotient topology on a set A, relative to a topological space X and a surjective map $X \to A$
- what it means for a continuous map $f : X \to A$ to be a quotient map, given topological spaces X, A
- examples where an equivalence relation on X gives rise to a quotient map $X \to X/{\sim}$

§23–25.

- how to check that $U, V \subseteq X$ form a separation of X, or that X is connected
- how connected subspaces of X interact with separations of X
- why a union of connected subspaces of X that have a point in common is again connected
- how connectedness interacts with continuous maps and finite products
- key properties used in the proof that $\mathbf{R}$ is connected
- why $\mathbf{Q}$ is totally disconnected as a subspace of (analytic) $\mathbf{R}$, but not discrete
- definition of path-connectedness
- why the topologist's sine curve is connected (even though it is not path-connected)
- definitions of connected components and path components
- what it means to be locally connected or locally path connected
- examples of spaces that are disconnected but locally connected

$\approx$ *§26–27.*

- open-covering definition of compactness
- how compactness interacts with continuous maps and finite products
- statement of Heine–Borel for $[0, 1]$

What We'll Have Covered by Then, But Will Not Appear.

- the evenly-spaced topology on **Z**
- the countable-complement topology
- the K-topology on **R**
- the axiom of choice
- inequivalent metrics that generate the same topology (Problem Set 2, #4)
- the intermediate value theorem
- the tube lemma (Munkres Lem. 26.8)
- various hard proofs (of how connectedness/compactness interact with finite products, of why the topologist's sine curve is not path-connected, of Heine–Borel, *etc.*)
- the "long line" (Munkres 158–159, #12)
- paracompactness and second countability
- manifolds

What We Won't Have Covered by Then.

- Munkres §14 (the order topology)
- Munkres §21 (further discussion of the metric topology)
- open and closed maps
- the "infinite broom" (Munkres 162–163, #7)
- quasicomponents (Munkres 163, #10)
- the extreme value theorem
- Lebesgue number
- uniform continuity
- Munkres §28 (limit point compactness, sequential compactness)
- Munkres §29 (local compactness)