MATH 430: INTRODUCTION TO TOPOLOGY
FINAL EXAM GUIDE

SPRING 2025

The final exam will be held **in Kline Tower (KT) 205, on Sunday, May 4, at 2:00 pm**. It will be a closed-book exam, designed to take < 120 minutes.

What Could Appear.

§12–13. Topologies.

- comparing the discrete, indiscrete, and finite-complement topologies on a set; also to the analytic topology on $\mathbf{R}^n$ and the lower-limit topology on $\mathbf{R}$
- what it means for a subbasis or basis to generate a given topology
- examples where different bases generate the same topology

§16–17, 22. Subspaces, Continuous Maps, Quotients.

- given $A \subseteq Y \subseteq X$, how the interior/closure of A in Y are related to the interior/closure of A in X
- relationship between closures and limit points
- what it means to check that a space is Hausdorff ($= T_2$) or T_1
- examples of spaces which are T_1 but not Hausdorff, or which are not T_1
- relationship between the Hausdorff axiom and limit points
- what it means to check continuity using a basis/subbasis of the target
- examples of continuous bijections that are not homeomorphisms
- the pasting lemma (Munkres Thm. 18.3)
- what it means for a continuous map $f : X \to A$ to be a quotient map, given topological spaces X, A
- examples of the quotient space $X/{\sim}$ arising from an equivalence relation $\sim$

§15, 19–20. Products, Metrics.

- characterization of the product topology in Munkres Thm. 19.6
- comparing the box and product topologies on $\mathbf{R}^\infty, \mathbf{R}^\omega$ (Problem Set 3)
- comparing the euclidean and square metrics on $\mathbf{R}^n$; the uniform metrics on $\mathbf{R}^\infty, \mathbf{R}^\omega$

§23–27, 36. Connectedness, Compactness, Manifolds.

- how connectedness interacts with subspaces, continuous maps, and finite products
- why totally disconnected is stricter than discrete
- why path-connected implies connected
- the intermediate value theorem
- examples of spaces that are not (path-)connected but locally (path-)connected
- how compactness interacts with continuous maps and finite products
- Heine–Borel for $[0, 1]$ and consequences for related spaces
- second countability and the definition of a manifold

§51–52, 54, 58–59. Fundamental Groups.

- comparing homotopy and path homotopy
- how (path) homotopy interacts with composition of maps ($\circ$)
- the definition of $\pi_1(X, x)$
- effect of changing the basepoint x on $\pi_1(X, x)$
- effect of (star-)convexity of X on $\pi_1(X, x)$
- key ideas in the proof that $\pi_1(S^n, x)$ is trivial for $n \geq 2$
- examples of deformation retracts
- effect of retract(ion)s and homotopy equivalences on homomorphisms of π_1's
- comparing homeomorphisms and homotopy equivalences

§68–70, 72–73. The Seifert–van Kampen Theorem.

- examples of subgroups, kernels, normal subgroups, quotients
- definition of reduced words and their lengths
- construction of free groups
- universal property of the free product $G_1 * G_2$ (Lem. 68.1)
- definitions and exampes of generating sets and group presentations
- statement of Seifert–van Kampen, especially the hypotheses
- $\pi_1(S^1 \vee S^1)$ and more general wedge products
- effect of attaching a 2-cell to a Hausdorff space (Munkres Thm. 72.1; see also §60)
- applications of Munkres Thm. 72.1 to the torus and dunce caps (Munkres §73) and to the Klein bottle

§53–54. Coverings.

- definitions of covering maps and covering spaces
- statement of the path-lifting and homotopy-lifting properties
- $\pi_1(S^1)$ and the key ideas in its computation
- examples of covering spaces of $S^1 \vee S^1$, tori, dunce caps, Klein bottles
- effect of covering maps on homomorphisms of π_1's
- statement of the lifting correspondence (Munkres Thm. 54.6, or my restatement of it from class)

What We'll Have Covered by Then, But Will Not Appear.

- various hard proofs (why the topologist's sine curve is not path-connected, why $\pi_1(X, x)$ satisfies the group axioms, the existence of free products of groups, the path/homotopy-lifting properties, the full calculation of $\pi_1(S^n)$ for $n > 0$, *etc.*)
- the K-topology on $\mathbf{R}$
- the axiom of choice
- the tube lemma (Munkres Lem. 26.8)
- the "long line" (Munkres 158–159, #12)
- commutator subgroups
- torsion subgroups of abelian groups
- direct sums of abelian groups and free abelian groups
- products versus coproducts

- effect of connect sum on π_1's of surfaces
- the Alexander horned sphere
- statement of the Galois correspondence
- pointed coverings and (pointed) equivalences of coverings
- the construction of a universal covering from the path space
- statement of the classification of compact surfaces

What We Won't Have Covered by Then.

- Munkres §5–8, 56–57, 75, 81–82 and Chapter 10
- the Brouwer fixed-point theorem (Munkres Thm. 55.6)
- the proofs of Heine–Borel for $[0, 1]$ and Seifert–van Kampen
- the formal definition of a group action
- the formal definition of labelling schemes (Munkres §74)
- the formal definitions of cutting and pasting (Munkres §76)
- proof of the Galois correspondence