18.704: SEMINAR IN ALGEBRA

SPRING 2022 SCHEDULE

Setup & Background.

1/31		Welcome & syllabus; classical Fourier analysis
2/2	**Ch. 2**, 30-37	Anatomy of a talk (*with* Susan Ruff); the discrete Fourier transform (DFT)
2/4	37-44	Properties of the DFT

Block 1. Spectral graph theory, finite Euclidean geometry, random walks.

2/7	**1**	**Ch. 3**, 52-54	The spectral theorem for self-adjoint operators
	2	49-51, 54-55	Graphs, adjacency matrices, graph spectra (Thm. 1)
2/9	**3**	58-61	Spectra of Cayley graphs on $\mathbb{Z}/n\mathbb{Z}$ (Thm. 2)
	4	55-56; **Ch. 4**, 70-72	Ramanujan graphs; the Cayley graph $X(\mathbb{Z}/n\mathbb{Z}, \pm 1)$
2/11	**5**	**Ch. 3**, 61-64	Classification of finite fields (Thm. 3)
	6	64-67	Galois theory of finite fields (Thm. 4), trace and norm; Winnie Li's graph (Ex. 1-2)
2/14	**7**	**Ch. 4**, 74-76	Winnie Li's graphs are Ramanujan (Ex. 3); Kloosterman sums
	8	77-78	Fan Chung's diameter estimate (Thm. 1)
2/16	**9**	**Ch. 5**, 83-86	Finite Euclidean spaces and graphs
	10	89-92	Generalized Kloosterman sums, Weil's bound (Thm. 1); Prove (13) and (17)
2/18	**11**	**Ch. 6**, 98-100	Markov chains, transition matrices; the random walk on a circle (Q. 1-2)
	12	102-104	The limiting distribution is uniform (Thm. 1)
2/21			**Buffer Day (Presidents' Day)**
2/23	**13**	104-106	An effective version of the limit theorem (Thm. 2)
	14	106-108	Ex. 1-4
2/25	**15**	**Ch. 7**, 114-116	Spectra of Laplacians
	16	116-119	The heat equation (Finite Heat Kernel #1)
			Choice of talk 4 topic due

Block 2. Finite abelian groups, binary codes, error correction, physics. **S** stands for the article by Sloane.

2/28	1	**Ch. 10**, 167-168	Characters of finite abelian groups, dual groups; the DFT on finite abelian groups (Thm. 2)
	2	172-175	Hadamard matrices, Walsh functions
3/2	3	175-177	Hamming graphs, weights, and distance (Ex. 1)
	4	178-181	Krawtchouk polynomials; symmetric spaces, spherical functions (Ex. 2)
3/4	5	**S**, 231, 233-236.	Association schemes, the Bose–Mesner algebra
	6	**S**, 238, 241-244.	Hamming and Johnson association schemes
3/7			**Buffer Day**
3/9	7	**Ch. 11**, 188-190	Binary codes, $[n, k]$- and $[n, k, d]$-codes, cyclic codes; polynomial rings and their ideals
	8	190-193	Examples of codes (Ex. 1-3)
3/11	9	193-195	Lloyd's theorem on perfect error-correcting codes; the classification of binary perfect $[n, k, d]$-codes
	10	**Ch. 12**, 197-199	Duality between subgroups and quotients (Lem. 1); classical Poisson summation
3/14	11	199-201	Discrete Poisson summation (Thm. 1, Cor., Lem. 2)
	12	202-204	Code form of Poisson summation; weight enumerators, the MacWilliams identity (Thm. 2)
3/16	13	**Ch. 13**, 215-217	Dynamics of vibrating springs
	14	218-219	Benzene
3/18	15	213-214	Ehrenfest's urn model of heat diffusion
	16	**Ch. 14**, 223-226	The discrete uncertainty principle (Thm. 1)
			Outline of paper sections due

Block 3. Nonabelian harmonic analysis, *i.e.*, representation theory of finite groups. **E** stands for the book by Etingof *et al.*

3/28			**Buffer Day**
			Anatomy of written exposition (*with* Susan Ruff)
3/30	1	**Ch. 15**, 240-241	The groups $\mathrm{GL}(n, \mathbb{F}_q)$, $\mathrm{Aff}(q)$, $\mathrm{Heis}(q)$, $\mathrm{SL}(n, \mathbb{F}_q)$
	2	242-243, 250 *also* **E**, §2.3, 4.4	Representations, their equivalence, subreps, irreps; sums, tensor products, duals
4/1	3	244-245	Unitarizability (Prop. 1)

		also **E**, §4.6	
	4	247-248 *also* **E**, §4.1	Maschke's theorem (Prop. 2)
4/4	**5**	248-249 *also* **E**, §2.3	Schur's lemma (Lem. 1); 1-dimensionality of irreps of abelian groups
	6	251-254 *also* **E**, §4.5	Schur orthogonality (Thm. 1, Cor.); multiplicity of an irrep
4/6	**7**	254-255 *also* **E**, §4.8	Character tables
	8	246-247, 256	The group algebra, the regular representation (Lem. 2)
4/8	**9**	257-258 *also* **E**, §4.7	The Peter–Weyl theorem (Thm. 2(1))
	10	258-261 *also* **E**, §4.2	Class functions (Lem. 3), properties of characters (Thm. 3)
4/11	**11**	258, 261-263	Properties of the Fourier transform (Thm. 2(3), 4)
	12	263-266 *also* **E**, §4.3, 4.8	More examples, *e.g.*, S_3, S_4, A_4, Dih_4, Q_8
4/13	**13**	**Ch. 16**, 267-270	Induced representations (Ex. 1-2)
	14	267-272	Frobenius's formula for induced characters (Prop. 1, Cor.)
4/15	**15**	272-275	Conjugacy classes and irreps of $\mathrm{Aff}(q)$ (Prop. 2, Table II.3)
	16	277-279	Frobenius reciprocity (Thm. 1); transitivity of induction (Prop. 4)
			Rough draft due to classmate and instructor

Block 4. $\mathrm{GL}(2, \mathbb{F}_q)$ and its subgroups. **PS** stands for the book by Piatetski-Shapiro.

4/18			**Buffer Day**
4/20	**1**	**Ch. 17**, 282-283	Mackey–Wigner's "method of little groups"; application to $\mathrm{Aff}(q)$
	2	283-286	Cayley graphs of $\mathrm{Aff}(q)$
4/22	**3**	288-291	Classical and discrete wavelets (Thm. 2 and/or 3)
	4	**Ch. 18**, 294-297	Conjugacy classes and irreps of $\mathrm{Heis}(q)$ (Table II.4)
4/25	**5**	299-300	The Schrödinger representation; radar cross-ambguity
	6	**Ch. 19**, 302-305	The real upper half-plane and its Laplacian
4/27	**7**	305-307	The finite upper half-plane
	8	307-310	Finite symmetric spaces; Gelfand's and Selberg's criteria (Thm. 1, Ex., Cor. 1)

4/29	**9**	**Ch. 8**, 132-136	Gauss sums as gamma functions; quadratic reciprocity(?)
	10	**Ch. 20**, 330-333	k-Bessel functions as eigenfunctions of the Laplacian
			Feedback on classmate's draft due
5/2	**11**	**Ch. 21**, 363	Exceptional isomorphisms for SL, PSL in rank 2
	12	364-366	Maximal tori; conjugacy classes in $\mathrm{GL}(2, \mathbb{F}_q)$ (Table II.11)
5/4	**13**	366-369	Construction of the principal series representations
	14	369-373 *also* **PS**, 33-34	Construction of the discrete series representations
5/6	**15**	373-374 *also* **PS**, 35-38	Characters of the discrete series representations; character table of $\mathrm{GL}(2, \mathbb{F}_q)$
	16	381-384	Character table of $\mathrm{GL}(3, \mathbb{F}_q)$ (sketch only)

Coda.

5/9	**Buffer Day**	
	Special bonus talk?	
	Final draft of paper due	