MATH 250: TOPOLOGY I EXAM 2 GUIDE

FALL 2026

Exam 1 will be held in-class on **Monday, October 19, 2026**. It will start shortly after 12:40 pm and end at 2:00 pm. It will be roughly five to six problems long.

You will be allowed to look at any notes on paper that you wrote prior to the exam. However, you will not be allowed to use electronic devices of any kind—including phones, computers, tablets, or audio devices—or any software, including AI tools.

What Could Appear

Products of Spaces.

- bases for the box and product topologies on an arbitrary product of spaces $\prod_{i \in I} X_i$
- how the box and product topologies compare to each other, in general
- how to characterize the product topology on $\prod_i X_i$ in terms of the projection maps $\mathrm{pr}_j \colon \prod_i X_i \to X_j$
- an example of a space A and map $f \colon A \to \mathbf{R}^\omega$ such that f is continuous relative to the product topology on $\mathbf{R}^\omega$, but not the box topology
- other possible bases for the box and product topologies on $\mathbf{R}^\omega$

Metric Spaces.

- how a metric induces a basis, hence a topology
- how to define convergence in topological spaces, and how it generalizes convergence in metric spaces
- why the Euclidean and square metrics on $\mathbf{R}^n$ are equivalent (in the sense of Problem Set 2, #2)
- how to check whether two different metrics generate the same topology
- a basis for the uniform topology on $\mathbf{R}^\omega$
- how the uniform topology compares to the box and product topologies on $\mathbf{R}^\omega$

Closure and Interior.

- the definition of the limit of a sequence of points in a topological space X
- the definition (in Munkres's sense) of the limit points of a subset $A \subseteq X$
- the definition of the interior, in X, of a subset $A \subseteq X$
- the definition of the closure, in X, of a subset $A \subseteq X$...
  - ...in terms of interior
  - ...in terms of the limit points of A
- examples where $\mathrm{Int}_A(A) \neq \mathrm{Int}_X(A)$ and $\mathrm{Cl}_A(A) \neq \mathrm{Cl}_X(A)$
- given $A \subseteq Y \subseteq X$, how $\mathrm{Cl}_Y(A)$ and $\mathrm{Cl}_X(A) \cap Y$ are related

- what the Hausdorff property implies about limits of sequences
- how to decide if a point of $\mathbf{R}^\omega$ belongs to the closure of $\mathbf{R}^\infty \subseteq \mathbf{R}^\omega$

Quotient Spaces.

- given a topological space X and map $p\colon X \to Q$, the definition of the quotient topology on Q inherited from p
- how the quotient topology on a set Q inherited from a map $p\colon X \to Q$ compares to other topologies making p continuous
- how the subspace topology on $S^1 \subseteq \mathbf{R}^2$ compares to the quotient topology it inherits from the cosine-sine map $[0,1] \to S^1$
- examples of describing a quotient space in the form $X/\!\sim$, for some equivalence relation $\sim$ on some space X
- examples of non-Hausdorff quotients of Hausdorff spaces

Practice Problems

Try to hand-write a solution to each problem within 10–15 minutes. Throughout, $\mathbf{R}$ has the **analytic** topology. On the exam, you will not need to use complete sentences, but the clearer your work, the more points you will earn.

Problem 1. Show that if X and Y are Hausdorff space, then so is $X \times Y$.

Problem 2. Let t be the taxicab metric on $\mathbf{R}^2$ defined by

$$t(\vec{u}, \vec{v}) = |u_1 - v_1| + |u_2 - v_2|.$$

Show that t is equivalent to the Euclidean metric.

Problem 3. Show explicitly that 0 is a limit point of $K = \{1/n \mid n = 1, 2, 3, \ldots\}$, whereas 1 is not.

Problem 4. (No need for full justifications.)

(1) Does $\mathbf{Z} \subseteq \mathbf{R}$ have limit points? if so, what are they?
(2) What is $\mathrm{Int}_{\mathbf{R}}(\mathbf{Z})$?
(3) What is $\mathrm{Cl}_{\mathbf{R}}(\mathbf{Z})$?

Repeat (1)–(3) with $\mathbf{Q}$ in place of $\mathbf{Z}$.

Problem 5. For any real numbers $a < b$, prove explicitly that $\mathrm{Int}_{\mathbf{R}}([a,b]) = (a,b)$. *Hint:* Handle the cases where $x \in (a,b)$ and where $x \in \{a,b\}$ separately.

Problem 6. Let $x = (1, 1/2, 1/3, \ldots) \in \mathbf{R}^\omega$. Decide whether or not x belongs to the closure of $\mathbf{R}^\infty$ in the uniform topology.

Problem 7. Let $\sim$ be the relation on $\mathbf{R}$ in which $x \sim y$ if and only if $|x| = |y|$.

(1) Show that $\sim$ is an equivalence relation.
(2) Show that $\mathbf{R}/\!\sim$ is homeomorphic to the subspace $[0, \infty) \subseteq \mathbf{R}$.

Problem 8. Let $p\colon \mathbf{R} \to S^1$ be the map $p(s) = (\cos(2\pi s), \sin(2\pi s))$. Show that the quotient topologies on S^1 induced by $p|_{[0,1)}$ and $p|_{[0,1]}$ are different.