MATH 250: TOPOLOGY I EXAM 1 GUIDE

FALL 2026

Exam 1 will be held in-class on **Monday, September 14, 2026**. It will start shortly after 12:40 pm and end at 2:00 pm. It will be roughly five to six problems long.

You will be allowed to look at any notes on paper that you wrote prior to the exam. However, you will not be allowed to use electronic devices of any kind—including phones, computers, tablets, or audio devices—or any software, including AI tools.

What Could Appear

Background from Analysis.

- why the Euclidean and discrete metrics are metrics
- definition of d-balls for a metric d
- definition of analytic continuity and analytically open sets

Topological Spaces.

- definition of a topology
- definitions of the discrete, indiscrete, and finite-complement (*a.k.a.* cofinite) topologies on any set, and why these are topologies
- definition of the analytic and lower-limit topologies on $\mathbf{R}$
- which of the above topologies on $\mathbf{R}$ are finer or coarser than others

Bases.

- what it means for a collection of subsets of X to be a basis
- what it means for a basis to generate a given topology on X
- examples where different bases generate the same topology on X

Continuous Maps.

- what it means for a map between topological spaces to be continuous
- how to check continuity of $f\colon X \to Y$ using a basis for the topology on Y
- what it means for topological spaces to be homeomorphic
- examples of homeomorphisms between distinct sets (*e.g.*, $\mathbf{R}$ and $(0,1)$)
- examples of continuous bijections that are not homeomorphisms

Subspaces.

- definition of the subspace topology on $A \subseteq X$, given a topology on X
- how the subspace topology on A is related to continuity of the inclusion map from A into X
- examples where some subset of A is open in A, but not in X

Finite Products of Spaces.

- definition of a direct product of sets $\prod_{i \in I} X_i$
- definition of the product topology on a finite product of spaces
- how the product topology on $\prod_{i \in I} \colon X_i$ is related to continuity of the projection maps $\mathrm{pr}_j \colon \prod_{i \in I} X_i \to X_j$
- comparison of the analytic and product topologies on $\mathbf{R}^2 = \mathbf{R} \times \mathbf{R}$

Practice Problems

Try to hand-write a solution to each problem within 10–15 minutes. Throughout, $\mathbf{R}$ has the analytic topology unless otherwise specified.

On the exam, you will not need to use complete sentences, but the clearer your work, the more points you will earn.

Problem 1. Prove that the finite-complement topology on $\mathbf{R}$ is coarser than the analytic topology.

Problem 2. Let $\mathcal{B}$ be the collection of intervals in $\mathbf{R}$ of the form $[a, b)$ (where $a < b$). It turns out that $\mathcal{B}$ is a basis. Show that the topology it generates is strictly finer than the analytic topology.

Problem 3. Give spaces X and Y and a continuous bijective map $f : X \to Y$ that is not a homeomorphism. You do not need to prove that f is continuous or bijective, but you should prove that it is not a homeomorphism.

Problem 4. Let $\mathcal{T}, \mathcal{T}'$ be topologies on X, with $\mathcal{T}$ coarser than $\mathcal{T}'$.

(1) If $f : X \to Y$ is continuous for $\mathcal{T}$, must f be continuous for $\mathcal{T}'$? If not, give a counterexample.

(2) If $g : Z \to X$ is continuous for $\mathcal{T}$, must g be continuous for $\mathcal{T}'$? If not, give a counterexample.

Problem 5. Prove that if $\mathcal{B}$ is a basis for a topological space X, and $A \subseteq X$, then

$$\{A \cap B \mid B \in \mathcal{B}\}$$

is a basis for the subspace topology on A.

Problem 6. Give an infinite subspace of $\mathbf{R}$ whose (subspace) topology is discrete. Justify your answer.

Problem 7. Show that the formula

$$\eta(x, y) = \min\{1, |x - y|\}$$

defines a metric η on $\mathbf{R}$.