MATH 250: TOPOLOGY I FINAL GUIDE

FALL 2025

The midterm exam will be held in-class on **Monday, December 8, 2025**. It will start at 12:30 pm and end at 3:00 pm (**2.5 hours**).

You will be allowed to look at any notes on paper that you wrote prior to the exam, and at the textbook (Munkres, *Topology*, 2nd Ed.). However, you will not be allowed to use electronic devices of any kind—including phones, computers, tablets, or other visual/audio devices—or any software.

What Could Appear

§12–13. Topologies.

- definitions of the discrete, indiscrete, and finite-complement topologies on any set, and why these are topologies
- how the analytic topology on $\mathbf{R}^n$ compares to the topologies above
- what it means for a basis to generate a given topology on X
- examples where different bases generate the same topology on X

§16–18, 22. Continuous Maps, Subspaces, Quotients.

- examples where some subset of A is open in A, but not in X
- definitions of the interior and closure of a subset A of a given space X
- definitions of the Hausdorff ($= T_2$) and T_1 properties
- what it means for a sequence of points to converge to a given point
- what the Hausdorff property implies for convergence of sequences
- what it means for a map between topological spaces to be continuous, or more strongly, a homeomorphism
- how to check continuity of a map $f\colon X \to Y$ using a basis for the topology on Y
- examples of continuous bijections that are not homeomorphisms
- examples of quotient spaces (*e.g.*, constructed using equivalence relations)

§15, 19–20. Products, Metrics.

- why $\mathbf{R}^n$ and $\mathbf{R}^\omega$ are examples of direct products of sets
- how the box and product topologies compare to each other, for $\mathbf{R}^n$, $\mathbf{R}^\infty$, $\mathbf{R}^\omega$
- how the product topology on $\prod_{i \in I} X_i$ is related to continuity of the various projection maps $\mathrm{pr}_j\colon \prod_{i \in I} X_i \to X_j$
- how the euclidean and square metrics compare to each other on $\mathbf{R}^n$
- why the uniform metric on $\mathbf{R}^\omega$ is a metric

§23–27. Connectedness and Compactness.

- how connected subspaces of X interact with separations of X
- how connectedness interacts with continuous maps and finite products
- spaces that are totally disconnected but not discrete
- why path-connected implies connected
- statement of the intermediate value theorem
- definitions of connected components and path components
- what it means to be locally connected or locally path connected
- examples of spaces that are disconnected but locally connected
- statement of Heine–Borel for subsets of $\mathbf{R}$

§51–52, 54, 58–59. Homotopy, Path Homotopy, Fundamental Groups.

- how homotopy and path homotopy differ
- how homotopies and path homotopies interact with compositions of maps
- why star-convex subsets of $\mathbf{R}^n$ are contractible
- examples of spaces that are homotopy equivalent, but not homeomorphic
- the definition of $\pi_1(X, x)$
- meaning of $f_* \colon \pi_1(X, x) \to \pi_1(Y, f(x))$ for a continuous map $f \colon X \to Y$
- effect of changing the basepoint x on $\pi_1(X, x)$
- an explicit isomorphism $\pi_1(S^1, p) \simeq \mathbf{Z}$ (say, where $p = (1, 0) \in \mathbf{R}^2$)
- $\pi_1(\prod_i X_i, (x_i)_i) \simeq \prod_i \pi_1(X_i, x_i)$

§68–71, ≈ 73. The Seifert–Van Kampen Theorem.

- meaning of a presentation of a group by generators and relations
- meaning of the free product $G_1 * G_2$ for groups G_1, G_2
- $\pi_1(S^1 \vee S^1)$ and similar wedge products
- definition of the homomorphism $\pi_1(U, x) * \pi_1(V, x) \to \pi_1(X, x)$, when $X = U \cup V$ and $x \in U \cap V$
- examples where $X = U \cup V$ and $x \in U \cap V$, but $\pi_1(X, x) \not\simeq \pi_1(U, x) * \pi_1(V, x)$
- statement of Seifert–van Kampen, especially the hypotheses

§53. Coverings.

- examples and non-examples of covering maps
- statement of the path-lifting and homotopy-lifting properties

What We'll Have Covered by Then, But Will Not Appear

- the axiom of choice
- equivalence/inequivalence of metrics (Problem Set 2, #10)
- convergence in the uniform topology on $\mathbf{R}^\omega$
- the topologist's sine curve
- regularity and normality
- computing π_1 of a genus-g surface, for $g \geq 2$
- computing π_1 of the real projective plane (*a.k.a.*, "dunce cap")