MATH 199/249: INTRODUCTION TO GENERAL TOPOLOGY
EXAM 2 GUIDE

FALL 2026

Exam 2 will be held in-class on **Tuesday, October 20, 2026**. It will start shortly after 5:10 pm and end at 6:30 pm. It will be roughly five to six problems long.

You will be allowed to look at any notes on paper that you wrote prior to the exam. However, you will not be allowed to use electronic devices of any kind—including phones, computers, tablets, or audio devices—or any software, including AI tools.

What Could Appear

Analytic Continuity and Open Sets.

- definition of (analytically) open balls in a metric space
- how to define analytic continuity in terms of open balls
- definition of open sets in a metric space
- how to prove that an open ball is, indeed, analytically open

Topologies, Topological Spaces.

- how to check that a collection of subsets of X is or is not a topology on X
- how to check if one topology is coarser than, finer than, or incomparable to another on the same set
- (many) examples of topological spaces

Bases.

- how to check that a collection of subsets of X forms a basis (*i.e.*, the two axioms)
- how to check that a particular collection $\mathcal{T}$ is the topology generated by a given basis $\mathcal{B}$
- how to check that a particular basis $\mathcal{B}$ generates a given topology $\mathcal{T}$
- an example where two different bases generate the same topology (on the same set)

Topological Continuity.

- how to check that a map of topological spaces, $f\colon X \to Y$, is continuous
- how taking preimages interacts with unions and intersections
- how the continuity of $f\colon X \to Y$ depends on the specific topologies chosen on X and Y
- how to check that a map is a homeomorphism

Subspaces.

- given a subset A of a topological space X, the definition of the subspace topology on A inherited from X
- examples where X is not discrete but $A \subset X$ is discrete
- other examples where some set $U \subseteq A$ is open in A, but not open in X
- an example where A is a proper subset of X, but subsets of A are open in A if and only if they are open in X
- an example where A is a proper subset of X, but homeomorphic to X

Closure and Interior.

- the definition of the limit of a sequence of points in a topological space X
- the definition (in Munkres's sense) of the limit points of a subset $A \subseteq X$
- the definition of the interior, in X, of a subset $A \subseteq X$
- the definition of the closure, in X, of a subset $A \subseteq X$...
  - ...in terms of interior
  - ...in terms of the limit points of A
- examples where $\mathrm{Int}_A(A) \neq \mathrm{Int}_X(A)$ and $\mathrm{Cl}_A(A) \neq \mathrm{Cl}_X(A)$
- the definition of the Hausdorff property
- what the Hausdorff property implies about limits of sequences

Practice Problems

Try to hand-write a solution to each problem within 10–15 minutes. In Problems 6–10, assume that $\mathbf{R}$ has the **analytic** topology. On the exam, you will not need to use complete sentences, but the clearer your work, the more points you will earn.

Problem 1. Prove explicitly that the set $(11, 99)$ is analytically open in $\mathbf{R}$. That is, for each x in this set, give an explicit $\delta > 0$ such that $B(x, \delta)$ is contained within the set.

Problem 2. Prove that $\mathbf{Z}$ is analytically closed in $\mathbf{R}$. That is, prove that its complement is open.

Problem 3. Let $f\colon \mathbf{R} \to \mathbf{R}$ be the function $f(x) = x^2 - 1$. Find the preimages under f of the following subsets of the range:

(1) $(0, 3)$.
(2) $[-1, 1]$.
(3) $[-2, -1]$.
(4) $(-\infty, 0)$.

Problem 4. Write down topologies $\mathcal{T}_1, \mathcal{T}_2, \mathcal{S}$ on the set $\{a, b, c\}$ such that $\mathcal{T}_1$ is coarser than $\mathcal{T}_2$, and $\mathcal{S}$ is incomparable with $\mathcal{T}_2$. (No need to write down full justifications.)

Problem 5. Let X, Y be topological spaces and $f\colon X \to Y$ a map. Suppose that for any closed $Z \subseteq Y$, we know that $f^{-1}(Z)$ is closed in X. Show that f is continuous.

Problem 6. Le $g\colon \mathbf{R} \to \mathbf{R}$ be the floor function: $g(x)$ is the largest integer less than or equal to x. Using only the topological definition of continuity, show that g is not continuous. *Hint:* Compute $g^{-1}((-1,1))$.

Problem 7. Show that if $S \subseteq \mathbf{R}$ is a finite set, then the subspace topology that S inherits from $\mathbf{R}$ is discrete.

Problem 8. Show that if $A \subseteq Y \subseteq X$, then the subspace topologies that A inherits from X and from Y are the same.

Problem 9. (No need for full justifications below.)

(1) Does $\mathbf{Z} \subseteq \mathbf{R}$ have limit points? if so, what are they?
(2) What is $\mathrm{Int}_{\mathbf{R}}(\mathbf{Z})$?
(3) What is $\mathrm{Cl}_{\mathbf{R}}(\mathbf{Z})$?

Repeat (1)–(3) with $\mathbf{Q}$ in place of $\mathbf{Z}$.

Problem 10. For any real numbers $a < b$, prove explicitly that $\mathrm{Int}_{\mathbf{R}}([a,b]) = (a,b)$. *Hint:* Handle the cases where $x \in (a,b)$ and where $x \in \{a,b\}$ separately.